On the Measurement of the Complex Permittivity of Materials by an Open-Ended Coaxial Probe

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***Abstract*—A number of papers have appeared on the characterization of materials using a coaxial probe. In most cases, approximate admittance models have been used for this purpose. The present work compares two rigorous mathematical models for the aperture admittance. One of these relations is based on the variational principle while the other utilizes the method of moments. Muller's method is then used to determine the complex permittivity of materials. The two procedures have been tested in the frequency range of 1–18 GHz.**

I. Introduction

An OPEN-ENDED coaxial line method for determining the complex permittivity has received attention of many researchers because of its simplicity. This technique requires an accurate measurement of the admittance of coaxial opening and its precise relation with the dielectric properties of the terminating sample. A number of relations have been reported for the aperture admittance with the material characteristics. However, only simplified approximate expressions are used to determine the dielectric properties because of the complexity in the inversion process. Most of these relations are derived from a variational expression for the aperture admittance under certain assumptions and the measurement accuracy is improved via a suitable calibration procedure [1], [2]. An alternate method reported recently uses a polynomial representation that still requires some means to evaluate its parameters [3]. In the present work, we used the variational expression for the aperture admittance without simplifying it further and the Muller's method to locate the complex zeros. We have also used a full-wave formulation and the method of moments in place of the variational expression. The results obtained by these two procedures are verified with several materials. However, only selected results will be given here for the sake of brevity.

II. Theoretical Background

Consider a coaxial line with an infinite conducting ground plane over its open end. The medium over the ground plane is linear, isotropic, homogeneous, and nonmagnetic in nature. After assuming that only the principal mode fields are present at the opening, a variational expression for its aperture admittance is found as follows [1]:

$$Y_L = \frac{j2k^2}{\omega\mu_o\left[\ln\left(\frac{b}{a}\right)\right]^2}\int_a^b\int_a^b\int_0^\pi \cdot \cos(\phi')\frac{\exp(-jkr)}{r}\,d\phi'\,d\rho'\,d\rho \qquad (1)$$

where 'a' and 'b' are its inner and outer radii, respectively. A cylindrical coordinate system (ρ, ϕ, z) is used with prime coordinates representing the source point and the unprimed representing the field points. ω is the angular frequency; μ_o and ϵ_o are the permeability and permittivity of the free space, respectively. k is the wavenumber in material medium, and

$$r = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi')}. \qquad (2)$$

As mentioned above, it is assumed in formulating (1) that only the TEM mode fields are present over the coaxial opening. However, these fields can be determined accurately by solving the following integral equation for the radial component of electric field, $E_\rho(\rho', 0)$, over the aperture [4]:

$$\frac{\omega\mu_o}{\rho} + j\pi k_l^2\int_a^b E_\rho(\rho',0)K_c(\rho,\rho')\rho'\,d\rho' = jk^2\int_a^b\int_0^\pi \cos(\phi')E_\rho(\rho',0)\frac{\exp(-jkr)}{r}\rho'\,d\rho'\,d\phi' \qquad (3)$$

where,

$$K_c(\rho,\rho') = j\sum_{n=0}^{\infty}\frac{\phi_n(\rho)\phi_n(\rho')}{A_n^2\beta_n} \qquad (4)$$

$$\phi_n = Y_o(\gamma_n a)J_1(\gamma_n\rho) - J_o(\gamma_n a)Y_1(\gamma_n\rho) \qquad (5)$$

$$\beta_n = \begin{cases}\sqrt{k_l^2-\gamma_n^2} & k_l > \gamma_n\\ -j\sqrt{\gamma_n^2-k_l^2} & k_l < \gamma_n\end{cases} \qquad (6)$$

$$A_n^2 = \frac{2}{\pi^2\gamma_n^2}\left[\frac{J_o^2(\gamma_n a)}{J_o^2(\gamma_n b)} - 1\right] n > 0;$$

$$A_o^2 = \ln\left(\frac{b}{a}\right). \qquad (7)$$

The eigenvalues γ_n are solutions to the following characteristic equation:

$$J_o(\gamma_n b)Y_o(\gamma_n a) = J_o(\gamma_n a)Y_o(\gamma_n b). \qquad (8)$$

J_n and Y_n are Bessel functions of the first and second kind of order n, respectively. k_l is the wavenumber inside the coaxial line. A unit current excitation is assumed in this formulation.

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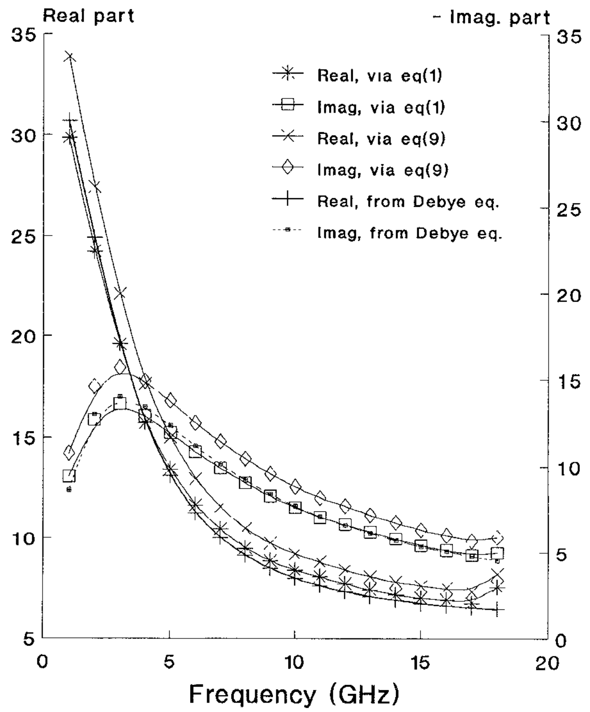

Fig. 1. Complex relative permittivity of methanol at room temperature.

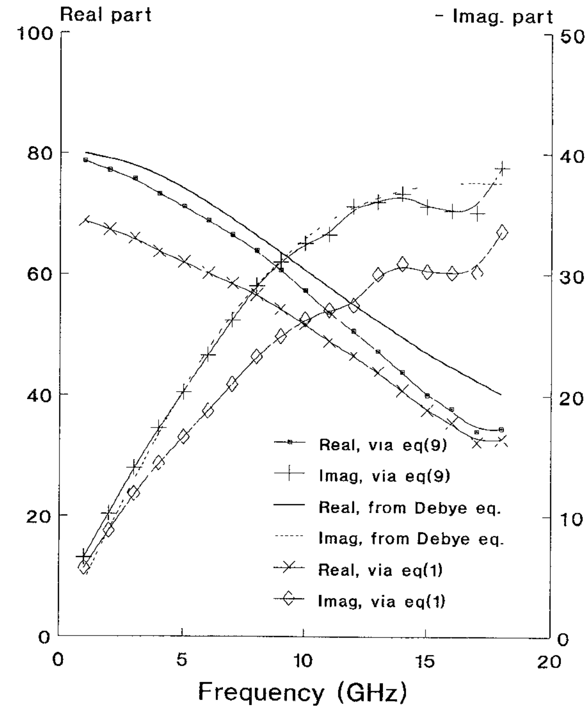

Fig. 2. Complex relative permittivity of water at room temperature.

Equation (3) is solved for the radial electric field $E_\rho(\rho', 0)$ using the method of moments. The aperture admittance, in turn, is determined from the following relation:

$$Y_L = \frac{2}{\int_a^b E_\rho(\rho', 0)\, d\rho'} - \frac{2\pi}{\left[\sqrt{\frac{\mu_o}{\varepsilon_o \varepsilon_l}} \ln(b/a)\right]}. \quad (9)$$

Equations (1) or (9) can be used to determine the wavenumber of the signal in terminating medium. In that case, the admittance Y_L in the left hand side of (1) or (9) is known from the measurements, and the equation can be cast in the form $f(k) = 0$. The wavenumber 'k' is evaluated as a root of this equation using the Muller's method. This iterative process requires three initial guesses k_{n-2}, k_{n-1}, k_n, and the values of function $f(k)$ at those points. The next zero of the function is determined as follows [5]:

$$k_{n+1} = k_n - (k_n - k_{n-1})\left[\frac{2A}{B \pm \sqrt{B^2 - 4AC}}\right] \quad (10)$$

where,

$$A = (1+q) \cdot f(k_n) \quad (11)$$

$$B = (2q+1) \cdot f(k_n) - (1+q)^2 \cdot f(k_{n-1}) + q^2 f(k_{n-2}) \quad (12)$$

$$C = q \cdot f(k_n) - q(1+q) \cdot f(k_{n-1}) + q^2 \cdot f(k_{n-2}) \quad (13)$$

$$q = \frac{k_n - k_{n-1}}{k_{n-1} - k_{n-2}}. \quad (14)$$

The '+' or '−' sign in the denominator of (10) is selected such that its magnitude is maximum.

III. Results

Technique described in the preceding section was tested with several materials. However, the results obtained only with methanol and water are presented here for the sake of brevity. Admittance data used in this section are available in the literature along with the experimental method [1]. It is to be noted that only factory-standard calibration loads were used to calibrate the ANA before collecting this data. Three initial values of 'k' to start iterations were picked arbitrarily by assuming the unknown complex permittivity as 1, 2, and 3. Fig. 1 illustrates the complex permittivity of methanol calculated from (1) as well as from (9). It also shows the corresponding data obtained from Debye relation [6]. While the general characteristics of these curves are similar, the results obtained from (9) show a little shift from the other two. For example, the permittivity determined from (1) is found to be 19.6 $-j13.6$ at 3 GHz against the corresponding result from Debye equation as 19.4 $-j13.7$. Equation (9) produces 22.1 $-j15.6$, which is a bit higher than the other two data. However, it is closer to a tabulated value of 23.9 $-j15.3$ [7].

Fig. 2 depicts the complex permittivity of water determined from (1), (9), and the corresponding Debye relation [6]. In this case, the results obtained from (1) are way off in comparison with the other two sets, namely, the Debye equation and (9). Slight differences between the latter two are attributed to experimental errors as well as the parameter data used in Debye relation. For example, the permittivity of water at 3 GHz is found to be 76.3 $-j10.9$ from the Debye equation. The corresponding results obtained via (1) and (9) are 66.1 $-j11.9$ and 75.8 $-j14$, respectively. Von Hippel has tabulated it as 76.7 $-j12$ [7], which is fairly close to the data obtained from (9). When the microwave frequency increases to 10 GHz,

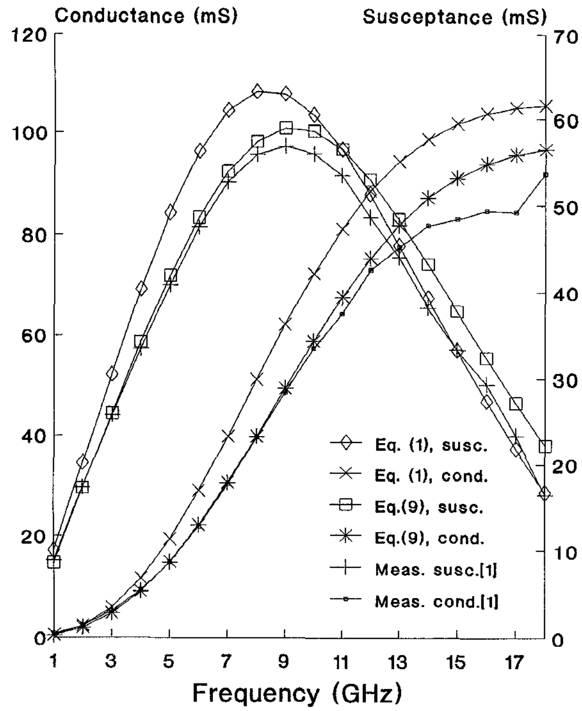

Fig. 3. Aperture admittance of a coaxial probe dipped in water (inner and outer radii 0.4 and 1.14 mm, respectively).

Debye relation produces a permittivity value as $63 - j29.5$. The corresponding values obtained from (1) and (9) are $51.7 - j26.2$ and $57.2 - j32.6$, respectively. This data tabulated by Von Hippel is $55 - j30$ [7], which is again very close to that obtained from (9). In order to analyze these results, the admittance values calculated from (1) and (9) are compared with the corresponding experimental results. The permittivity values needed for these computations were generated by the Debye equation. This comparison is illustrated in Fig. 3, which shows that the experimental results are fairly close to that obtained from (9). The corresponding characteristics obtained from (1) are somewhat displaced with respect to the other two. This displacement may be acceptable for the admittance evaluation but it affects seriously the inverse process of evaluating the complex permittivity. Another calibration scheme suggested in literature [1] also "corrects" for this discrepancy of the admittance model. However, it is not required with a more accurate admittance model as given by (9).

IV. Conclusion

Two different admittance models have been compared in this paper for measuring the complex permittivity of materials using a coaxial probe. The results indicate that the variational model works satisfactorily only for low permittivity materials. The admittance model based on a full-wave formulation and the method of moments produces accurate results even for high permittivity materials. However, it may take a few minutes to get an acceptable convergence against less than a minute with the former. The computations were performed on a 486DX4, 75 MHz notebook. It may be possible to reduce the computation time a bit further by optimizing the integration and algebraic routines used in implementing the method of moments.

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